De Broglie Wavelength Reduction for a Multi-photon Wave Packet

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An experiment is proposed that permits the observation of the reduced de Broglie wavelengths of two and four-photon wave packets using present technology. It is suggested to use a Mach-Zehnder setup and feed both input ports with light generated by a single non-degenerate down-conversion source. The strong quantum correlations of the light in conjunction with boson-enhancement at the input beam splitter allow to detect a two- and fourfold decrease in the observed de Broglie wavelength with perfect visibility. This allows a reduction of the observed de Broglie wavelength below the wavelength of the source.

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Recently Jacobson *et al.* [1] gave a theoretical description of how to measure the effective de Broglie wavelength of a quantum system in a coherent many-particle state using a scheme employing non-linear beam splitters. Their work concluded that many-particle systems with a sufficiently sharply defined number of constituents, in the best case prepared in a pure number state $|N\rangle$, should show a reduction from the single particle wavelength λ to the effective multi-particle wavelength λ/N. This is the case even if the constituents are not bound together. For bound systems such as poly-atomic molecules [2,3] this fact is well known, but for unbound systems such as N-photon Fock states $|N\rangle$ this finding is perhaps surprising.

The scheme suggested by Jacobson *et al.* uses non-linear beam-splitters in order to emulate an effective binding of the particles. In a two-mode interferometer this input beam-splitter channels the unbound constituents of the system such that they simultaneously follow the 'upper' interferometric channel u in superposition with all of them passing through the 'lower' channel l; the quantum state inside the interferometer reads

$$|\psi\rangle_{inside} = \frac{|N\rangle_u|0\rangle_l + |0\rangle_u|N\rangle_l}{\sqrt{2}} . \qquad (1)$$

Compared to the case of a bound system this state obviously corresponds to an N-atomic molecule traversing the interferometer undivided, thus maximally shrinking the system's de Broglie wavelength.

The output beam mixer of the interferometer, see Fig. 1, is supposed to be of the same non-linear kind as the input beam-splitter. Unfortunately, the use of non-linear beam-splitters amounts to a considerable practical problem with regards to the experimental implementation of the ideas presented in reference [1]: at present, neither the required large magnitudes of the non-linearities nor their different orders for different particle number N are practically achievable.

Inspired by Jacobson's work, but following a rather different approach, Fonseca *et al.* managed to measure the predicted halving of the wavelength for a two-photon state generated in parametric down-conversion [4]. The down-conversion source allowed them to abandon the non-linear beam-splitter in the input of their Young's double-slit setup, and a two-particle coincidence detection scheme allowed them to get rid of the second non-linear beam mixer. Here, I want to present a modification to their approach (borrowing ideas from [5,6] and [7]) that should allow for the observation of a fourfold reduction of the de Broglie wavelength with currently available technology [6,8].

The key idea is to use linear beam-splitters in a Mach-Zehnder setup and utilize boson-enhancement to generate good approximations to the ideal state (1) inside the interferometer. Combined with suitable post-selection of detection events this approach, in principle, even allows for the observation of more than fourfold reductions in the measured de Broglie wavelength of unbound multi-particle systems.

So far, only variations with the wavelength of the pump-beam, i.e. halving of the effective de Broglie wavelength (using one photon pair) have been observed [4,9–11], consequently the idea presented here will allow for a test of a quantum effect not observed before, namely, a reduction of the multi-particle de Broglie wavelength below that of the generating source [7,12]. See reference [13], however, for the observation of a quartering of the joint phase of the internal degrees of freedom of four entangled ions in a trap.

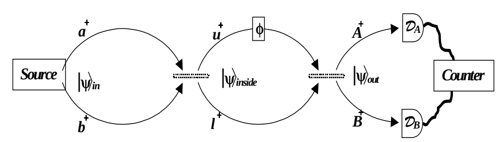

FIG. 1. Sketch of the interferometer: dotted lines outline balanced beam splitters, $\mathcal{D}$ stands for detectors and a, b, u, l, A and B label the modes before, inside and beyond the interferometer. ϕ is the phase shifter in the upper channel.

From now on we will only discuss photons passing a Mach-Zehnder interferometer, see Fig. 1. Implicitly this discussion covers many bosonic systems and all types of two-mode interferometers.

Rather than using non-linear beam splitters, even for two-photon states, it is easier to prepare the desired interferometric state (1) from photon-pairs generated in type II (non-degenerate) parametric down-conversion. Fonseca *et al.* used a pump beam with a momentum characteristic such that both photons would always pass the same slit [4]. Hence, the state $(|1,1\rangle_u|0,0\rangle_l + |0,0\rangle_u|1,1\rangle_l)/\sqrt{2}$ was created, where the two slots $|\cdot,\cdot\rangle$ refer to the two orthogonal polarization modes of the photons and the indices u and l refer to either slit in their double slit setup. This state is very similar to the desired form (1) and works just as well, as long as interferometric phase changes apply to both polarization modes simultaneously. Experiment [4] confirmed some ideas presented in [1] and earlier results about joint measurements on down-converted photons [9,10].

Another recent proposal [7] suggests to use a Mach-Zehnder layout and a projective measurement involving a third auxiliary photon (from a second down-conversion photon pair) in order to prepare state (1) for one photon pair inside the interferometer.

But both these approaches do not scale favourably for higher photon numbers, in the latter case [7] the use of auxiliary photons leads to extra channels and losses in the interferometer. In the former case of experiment [4] the momentum selection trick cannot be extended to higher photon numbers because multiple pump photons would be involved; this leads to a multiple product of the single-pair state mentioned above, i.e., in general to a state very different from the desired form (1). Therefore, another ingredient is needed to channel the photons through the interferometer, for this, boson-enhancement can be used.

As is known from the Mandel-dip experiment [14,15,7] the preparation of state (1) using boson-enhancement is straightforward for two photons: spontaneous parametric down-conversion generates single and multiple photon pairs in a two-mode squeezed vacuum [16] from the vacuum state $|0\rangle$

$$|\psi\rangle_{in} = \frac{1}{\sqrt{1-|\alpha|^2}} \sum_{n=0}^{\infty} \frac{(\alpha\, a^\dagger b^\dagger)^n}{n!}\, |0\rangle \ , \qquad (2)$$

here $a^\dagger$ and $b^\dagger$ are the bosonic creation operators for the field modes of the interferometer input ports, see Fig. 1. With probability $|\alpha|^2/(1-|\alpha|^2)$ a suitable array of lossless detectors [17], operating in coincidence, will detect two photons. This allows us to retrodict that state (2) became projected into state $|1,1\rangle$ which entered the interferometer and became converted into the state

$$|\psi\rangle_{inside} = \frac{|2\rangle_u|0\rangle_l + |0\rangle_u|2\rangle_l}{\sqrt{2}} \ , \qquad (3)$$

where the transformations for a balanced beam-splitter

$$a^\dagger = (u^\dagger + i\, l^\dagger)/\sqrt{2}, \quad b^\dagger = (u^\dagger - i\, l^\dagger)/\sqrt{2} \qquad (4)$$

have been assumed [15]. The behaviour of such photon pairs has been studied intensely [4,10,11,14].

For four and more photons, our scheme only allows us to prepare states similar to the ideal state (1). Let us assume that a fourfold coincidence has been detected (see [6,8]), this occurs with probability $|\alpha|^4/(1-|\alpha|^2)$. It allows us to infer that the incident state was of the form $|\psi\rangle_{in} = a^{\dagger 2} b^{\dagger 2}|0\rangle/2$. Inside the interferometer this becomes [6] ($\varepsilon \equiv 1$)

$$|\psi\rangle_{inside} = \left(\frac{u^{\dagger\,4}}{8} + \frac{u^{\dagger\,2} l^{\dagger\,2}}{4} + \frac{l^{\dagger\,4}}{8}\right)|0\rangle \qquad (5)$$

$$= \sqrt{\frac{3}{4}}\left(\frac{|4\rangle_u|0\rangle_l + |0\rangle_u|4\rangle_l}{\sqrt{2}}\right) + \frac{\varepsilon}{\sqrt{4}}\,|2\rangle_u|2\rangle_l \ . \qquad (6)$$

Obviously the bosonic enhancement leads to the generation of the desired state of the form (1) in 75% of all cases [6] whereas the unwanted contribution $|2\rangle_u|2\rangle_l$ only occurs in a quarter of all cases ($\varepsilon \equiv 1$). Although only an approximation to our goal, this state is readily available and the desired fourfold reduction of the de Broglie wavelength can be detected using current technology [6,8].

In order to determine the detectors' response we will now assume that the photons following channel u are delayed by a tunable phase ϕ and are subsequently mixed with the l-channel to form the detector modes $A^\dagger$ and $B^\dagger$:

$$u^\dagger = e^{i\phi}\,(A^\dagger + i\,B^\dagger)/\sqrt{2}, \quad l^\dagger = (A^\dagger - i\,B^\dagger)/\sqrt{2}\ , \qquad (7)$$

see Fig. 1. For the case of a single photon entering the interferometer through mode $a^\dagger$ we thus receive the final state

$$|\psi\rangle_{out} = \frac{(i + e^{i\phi})|1\rangle_A|0\rangle_B + (1 + i\,e^{i\phi})|0\rangle_A|1\rangle_B}{2} \qquad (8)$$

and the well known classical photo-detector response probabilities

$$P_A(\phi) = \langle A^\dagger A\rangle = \frac{1}{2}\,(1+\sin\phi) \qquad (9)$$

$$\text{and} \quad P_B(\phi) = \langle B^\dagger B\rangle = \frac{1}{2}\,(1-\sin\phi)\ . \qquad (10)$$

For the two-photon state (3) the corresponding expressions reflect the halving of the de Broglie wavelength

$$P_{AA}(\phi) = P_{BB}(\phi) = \frac{1}{4}\,(1+\cos 2\phi) \qquad (11)$$

$$\text{and} \quad P_{AB}(\phi) = \frac{1}{2}\,(1-\cos 2\phi)\ . \qquad (12)$$

Here P_{AA}, P_{BB} and $P_{AB} \equiv P_{BA}$ stand for the probabilities to detect two photons in the channels and with the multiplicity indicated by the subscripts. Since it is difficult to detect single photons and discriminate one from two photons arriving at the same time, the experimentally most convenient signal is P_{AB}.

The four-photon state (6) shows the expected reduction to a quarter of the de Broglie wavelength, namely

$$P_{AAAA}(\phi) = P_{BBBB}(\phi) = \frac{9 + 12\cos 2\phi + 3\cos 4\phi}{64} \ , \tag{13}$$

$$P_{AAAB}(\phi) = P_{ABBB}(\phi) = \frac{3 - 3\cos 4\phi}{16} \ , \tag{14}$$

$$\text{and} \quad P_{AABB}(\phi) = \frac{11 - 12\cos 2\phi + 9\cos 4\phi}{32} \ . \tag{15}$$

Surprisingly, despite the imperfect form ($\varepsilon = 1$) of the four-photon state (6), P_{AAAB} and P_{ABBB} show a pure fourfold reduction of the observed de Broglie wavelength with perfect visibility. Here P stands for the four-photon coincidence probabilities with the channels and their respective detection multiplicity indicated by the subscripts.

Since it is presently difficult to detect with single photon resolution and discriminate one from two or more photons arriving at the same time a special detector setup might have to be used. One can employ multi-port detectors as they are described in [18], a four-port in channel A suffices to see the signals P_{AAAA} and P_{AAAB} since it can split up four photons to follow four different channels, for sketches of possible realizations of four-port detectors see e.g. [6].

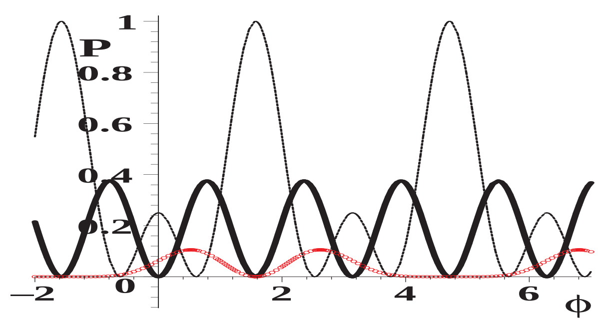

FIG. 2. The four-photon coincidence signals as a function of phase delay ϕ: P_{AAAB} (fat line) and P_{AABB} (thin line) in comparison with the classical signal $P_A P_A P_A P_B$ from eq. (10) (dotted red line).

For photon numbers $2n$ higher than four no pure $2n$-fold wavelength reduction like in P_{AAAB} is attainable using our scheme; also the problems with the generation and coincidence detection of more than four photons will make the experiment much harder to perform, we therefore will not write down the corresponding photodetection probabilities for 6, 8 or more photons.

Speculation: if one could, however, synthesize an input state of the form

$$|\psi\rangle_{in} = \frac{(a^\dagger - b^\dagger)^{2n} + (a^\dagger + b^\dagger)^{2n}}{\sqrt{2^{2n+1} \cdot 2n!}} |0\rangle \ , \tag{16}$$

where a and b stand for any two modes, say polarization, this state would, using the beam splitter operation (4), transform into a "Schrödinger-kitten state" (1)

$$|\psi\rangle_{inside} = \frac{|2n\rangle_u |0\rangle_l + |0\rangle_u |2n\rangle_l}{\sqrt{2}} \tag{17}$$

and would therefore also yield detector signals with perfect $2n$-fold wavelength reduction for any photon number $2n$. However, to my knowledge it is not known how to generate such a state with available technology. Yet, this observation stresses once more the alternatives to employing non-linear beam splitters [1] in order to channel the particles through the interferometer.

To conclude, interference patterns of unbound particles with halved de Broglie wavelengths have been seen in parametric down-conversion. Here a new scheme, using current technology, is proposed which allows to see further de Broglie wavelength reductions for such unbound particles below the wavelength of the generating source.

At the level of four photons generated in parametric down-conversion a fourfold reduction for the interference signal with perfect visibility is achievable.

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